EFFECTS OF INTERNAL HEAT GENERATION AND SUCTION/INJECTION ON A CHEMICALLY REACTING MHD HEAT AND MASS TRANSFER FLUID FLOW OVER A PERMEABLE SURFACE WITH CONVECTIVE BOUNDARY CONDITIONS

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Abstract: This work investigates the effects of internal heat generation and suction or injection parameter on a chemically reacting heat and mass MHD fluid flow over a permeable surface with convective boundary conditions. By transforming the system of coupled partial differential equations governing the flow into a system of coupled ordinary differential equations using similarity transformation, the resulting ordinary differential equations are solved using the Fourth order Runge-Kutta method with shooting technique. Increase in fluid suction retards rate of transportation but decrease the boundary layer thickness and vice versa for fluid injection. Internal heat generation increases fluid temperature and thickening of the thermal boundary layer.

Keywords: Internal heat generation, suction/injection, MHD heat & mass transfer

Introduction

The concept of boundary layer was first introduced by Prandtl in 1904 and since then it has been applied to several fluid flow problems. A boundary layer is formed whenever there is a relative motion between the boundary and the fluid. The details of flow within the boundary layer are very important for the understanding of many problems in aerodynamics, including the wind stall, the skin- drag on an object, heat transfers that occur in high speed flight and in naval architecture for the designs of ships and submarines. Bhattacharya *et al.* (2011) studied the slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet. Chaudary and Kumar (2013) studied the stagnation point flow and heat transfer for an electrically conducting fluid over a permeable surface in the presence of a magnetic field where the fluid was acted upon by an external uniform magnetic field and a uniform injection or suction which was directed normal to the plane of the wall.

Christian and Yakubu (2014) examined the effects of thermal radiation on magneto-hydrodynamic flow over a vertical plate with convective surface boundary condition but not for a chemically reactive fluid. Crane (1970) examined the problem of laminar boundary layer flow which arose from the flow of an incompressible viscous fluid past a stretching sheet in which the velocity near the stagnation point is proportional to the distance. Okedayo *et al.* (2012) presented similarity solution to the plane stagnation point flow with convective boundary conditions and obtained global Biot numbers. Adeniyan and Adigun (2013) considered the same problem under the influence of a uniform magnetic field which was placed transversely to the direction of the fluid flow. Aziz (2009) considered the classical problem of hydrodynamic and thermal boundary layer over a flat plate in a uniform stream of fluid using a similarity solution. Bhattacharya and Gupta (1985) established the stability of the mass and heat transfer for the boundary layer over a stretching sheet subject to suction or blowing which had first been looked at by Gupta and Gupta (1977). The same problem was also considered by Mahapatra and Gupta (2002) for different stretching and free stream velocities. Ishak (2010) considered a steady laminar boundary layer flow over a permeable flat plate in a uniform free stream with bottom surface of the plate heated by convection from a hot fluid using a similarity solution. Jat and Abhishak (2012) studied the flow and heat transfer for electrically conducting fluid over a non-linear stretching sheet.

The case for a fluid undergoing chemical reaction was considered by Emmanuel *et al.* (2014). Makinde and Olanrewaju (2010) analysed the effects of thermal buoyancy on the Laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective surface boundary condition using Shooting method. Olanrewaju *et al* (2011) analysed the effects of internal heat generation, thermal radiation and buoyancy force on the Laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective boundary condition with the assumption that the left surface of the plate is in contact with the hot fluid and a stream of cold fluid flow steadily over a right surface; the heat source decays exponentially towards the surface of the plate by using Shooting technique. Fenuga *et al.* (2015) considered the effects of buoyancy force and fluid injection/suction on a chemically reactive MHD flow with Heat and Mass Transfer over a permeable surface in the presence of Heat source/sink. Sajid and Hayat (2008) considered the effect of radiation on the boundary layer flow and heat transfer of a viscous fluid over an exponentially stretching sheet using Homotopy Analysis Method. This paper extends the work of Emmanuel *et al.* (2014) together with Fenuga *et al.* (2015) to include internal heat generation and sucking/injection on a chemically reacting MHD heat and mass transfer fluid flow over a Permeable surface with convective boundary conditions. The problem will be solved numerically using Runge-Kutta fourth order method with shooting technique.

NOMENCLATURE/ABBREVIATION

MHD	Magnetohydrohynomics
(x, y)	Coordinate axes or variables
(u, v)	Velocity components along the x-and y-axes
ψ	Dimensionless stream functions
g	Acceleration due to gravity
β_0	Magnetic Field Strength
T	Uniform Surface Temperature
T_∞	Far Stream Temperature or temperature at the boundary
T_f	Temperature at the plate surface
u_e	Free Stream Velocity
C	Concentration
C_∞	Far stream concentration or concentration at the boundary
C_f	Concentration at the plate surface

C_P	Specific heat capacity at constant pressure
γ	Coefficient of Kinematic viscosity
σ	Electrical Conductivity
K_P	Permeability of the medium
K	Thermal conductivity
D_m	Mass diffusivity
K_r	Reaction Rate constant
q_r	Radiative heat flux
Sc	Schmidt Number
P_r	Prandtl number
M	Magnetic parameter
G_r	thermal Grashof number
B_i	Biot number
β_c	Solutal expansion coefficient
β_τ	Thermal expansion coefficient
λ	Internal heat generation parameter
Ra	Radiation parameter
Da	Darcy parameter
B_r	Brinkman number
G_c	Solutal Grashof number
F_w	Suction or Injection parameter ((S > 0 for suction and S < 0 for injection)
B_s	Convective –diffusion parameter
δ	Slip parameter
(θ,ϕ)	Dimensionless temperature and concentration
Q	Heat release
h_m	Mass transfer
h_f	Heat transfer

Materials and Methods

Consider a two-dimensional, steady, hydrodynamic boundary layer flow with heat and mass transfer over a permeable uniformly moving plate placed in a saturated porous medium and the plane $y=0$ of a Cartesian coordinates system with the $x-axis$ along the surface of the plate in the presence of an externally applied magnetic field of constant strength $(0,\beta_0,0)$. The lower surface of the plate is heated by convection from a hot fluid at temperature T_f to give rise to a coefficient of heat transfer h_f. Also, the lower surface of the plate is heated by convection from a viscous fluid at concentration C_f to give rise to a coefficient mass transfer h_m. Furthermore the upper part of the plate is a Newtonian fluid which is electrically conducting as shown in Fig. 1.

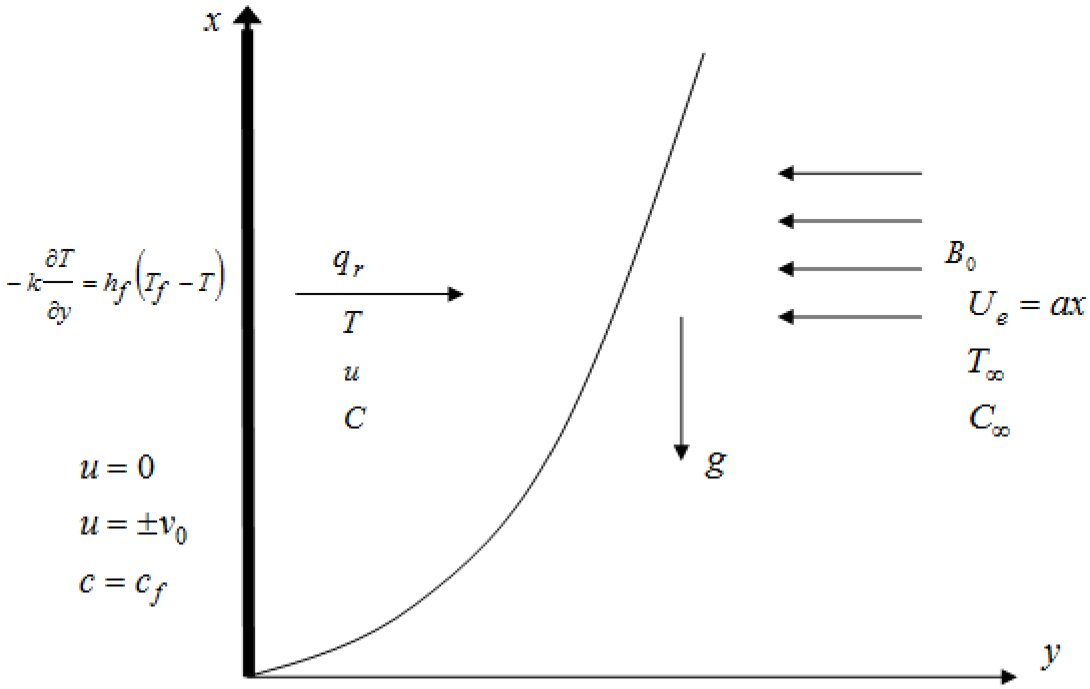

Fig. 1: Flow configuration and coordinate system

Using the Baussinesq approximation, the equations governing the fluid flow are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}+gB_T(T-T_\infty)-\frac{\sigma B_0^2(u-u_e)}{\rho}-\frac{\gamma}{K_P}(u-u_e)+gB_C(C-C_\infty) \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}+\frac{\gamma}{C_P}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2(u-u_e)^2}{\rho C_P}+\frac{Q}{\rho C_P}(T-T_\infty)-\frac{1}{C_P}\frac{\partial q_r}{\partial y} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-K_r(C-C_\infty) \quad (4)$$

The corresponding boundary conditions are

$$u(x,0)=L\frac{\partial u}{\partial y},\quad v(x,0)=\pm V_o \quad (5)$$

$$-K\frac{\partial T}{\partial y}(x,0)=\ h_f(T_f-T(x,0)) \quad (6)$$

$$-D\frac{\partial C}{\partial y}(x,0)=h_m(C_f-C(x,0)) \quad (7)$$

$$u(x,\infty)=u_e, T(x,\infty)=T_\infty, C(x,\infty)=C_\infty \quad (8)$$

Following Olanrewaju *et al.* (2011), the similarity variables are;

$$\eta=x^{-\frac{1}{2}}y\sqrt{\frac{u_e}{\gamma}},\qquad and \qquad \psi=x^{\frac{1}{2}}\sqrt{\gamma u_e}\,f(\eta)$$

Where: $u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x};\quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}\quad and\quad \phi(\eta)=\frac{C-C_\infty}{C_f-C_\infty}$ (9)

The u and v that satisfy equation (1) are

$$u=u_e f'(\eta)\quad \text{and}\qquad v=\frac{1}{2}\sqrt{\frac{u_e\gamma}{x}}(\eta f'(\eta)-f(\eta))$$

Using equation (9) to transforming the momentum, energy and concentration equations (1) to (4) together with the boundary conditions (5) to (9), we obtain;

$$f'''+\frac{1}{2}ff'+Gr_x\theta+Gc_x\phi+(M_x+Da_x)(1-f')=0$$

$$(1+\frac{4}{3}Bs_xRa)\theta''+\frac{1}{2}\Pr f\theta'+\Pr\lambda_x\theta+\Pr Ec(f'')^2+M_xEc\Pr(1-f')^2=0$$

..................(10)

$$\phi''+\frac{1}{2}S_cf\phi'-Sc\beta_x\phi=0$$

The corresponding boundary conditions are;

$$f'(0)=\delta_xf''(0);\ f(0)=F_{wx};\quad -\theta'(0)=Bi_x(1-\theta(0));$$
$$-\phi'(0)=Bs_x(1-\phi(0));\ f'(\infty)=1;\quad \theta(\infty)=0;\quad \phi(\infty)=0 \quad ....(11)$$

Where: the prime symbol represents the derivative with respect to η ,

$Sc=\frac{\gamma}{D_m}$ is the Schmidt number , $P_r=\frac{\rho C_p}{k}$ is the Prandtl number, $M_x=\frac{\sigma B_0^{\ 2}x}{\rho u_e}$ is the magnetic parameter,

$F_{wx}=\frac{\pm 2v_0x^{\frac{1}{2}}}{\sqrt{\gamma u_e}}$ is the injection/suction parameter ,

$Gr_x=\frac{xgB_\tau(T_f-T_\infty)}{u_e^{\ 2}}$ is the thermal Grashof number ,

$Gc_x=\frac{xgB_c(C_f-C_\infty)}{u_e^{\ 2}}$ is the solutal Grashof number ,

$\beta_x=\frac{k_rx}{u_e}$ is the reaction rate parameter, $\lambda_x=\frac{xQ}{u_e\rho c_p}$ is the internal heat generation parameter, $R_a = \frac{4\sigma * T_\infty^{\ 3}}{k'\rho C_p \gamma}$ is the radiation parameter , $Ec = \frac{u_e^{\ 2}}{C_p(T_f - T_\infty)}$ is the Eckert number, $Da_x \frac{x\gamma}{u_e k_p}$ is the Darcy parameter ,

$B_r = \frac{u_e^{\ 2}}{k(T_f - T_\infty)}$ is the Brinkman number,

$Bs_x = \frac{D}{h_m}\sqrt{\frac{u_e}{\gamma x}}$ is the convective-diffusion parameter ,

$Bi_x = \frac{h_f}{k}\sqrt{\frac{\gamma x}{u_e}}$ is the Biot number and $\delta_x = L\sqrt{\frac{u_e}{\gamma x}}$ is the slip parameter.

It is assumed that equations (10) with boundary conditions (11) have similarity solutions when the parameters M_x, F_{wx}, Gr_x , Gc_x, β_x , λ_x , Da_x, Bs_x, Bi_x and δ_x are defined as constants. The parameters of interest are Skin friction co-efficient $f''(0)$, plate surface temperature $\theta(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$.

Results and Discussion

Numerical calculations have been carried out for different values of the thermo physical parameters controlling the fluid dynamics in the flow region using Runge-Kutta method with shooting technique implemented by maple 17.

Table 1 shows the comparison of Emmanuel *et al.* (2014) with the present work for prandtl number P_r =0.72 and the work is in perfect agreement.

Table 1: Computations showing comparison of the present work with Emmanuel *et al.* (2014) for $n = 1$ and $M = Ra = Br = \beta = Bi = 0$

Profile							Emmanuel *et al.* (2014)		Present work	
$\Pr$	Sc	M	Ra	Br	β	Bi	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.71	0.24	0.1	0.1	0.1	0.1	0.1	0.451835	0.068283	0.451835005	0.06828319
0.72	0.24	0.1	0.1	0.1	0.1	0.1	0.451835	0.068415	0.451835005	0.06841531
0.72	0.24	0.1	0.1	0.1	0.1	0.1	0.770792	0.064224	0.770792288	0.0642241
0.72	0.24	0.1	0.1	0.1	0.1	0.1	0.451835	0.066984	0.451835005	0.0669827
0.72	0.24	0.1	0.1	0.1	0.1	0.1	0.451835	0.042658	0.451835005	0.04265801

Figures 2 to 6 explain the effects of emerging flow parameters on the velocity profile. In these Figures, the fluid velocity is lower at the plate surface and increases gradually to its free stream values satisfying the boundary condition. In Fig 4, an increase in the thermal Grashorf number leads to a thinning of the momentum boundary layer thickness and hence increasing the velocity of the flow. In Fig. 5, increase in fluid suction $F_W > 0$ retards the rate of transport and decreases the boundary layer thickness but vice versa for fluid injection $F_W < 0$.

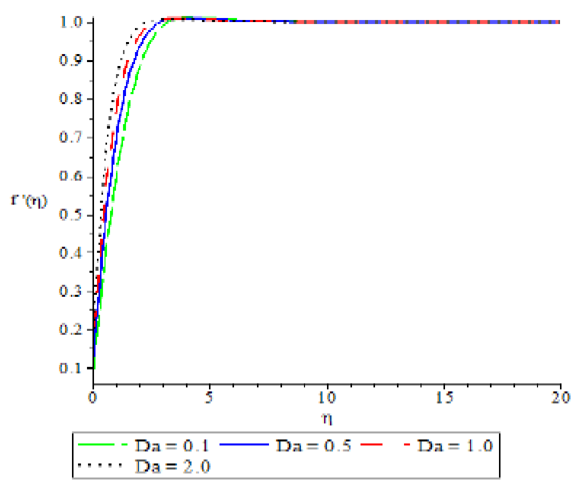

Fig. 2: Velocity profiles for varying Da

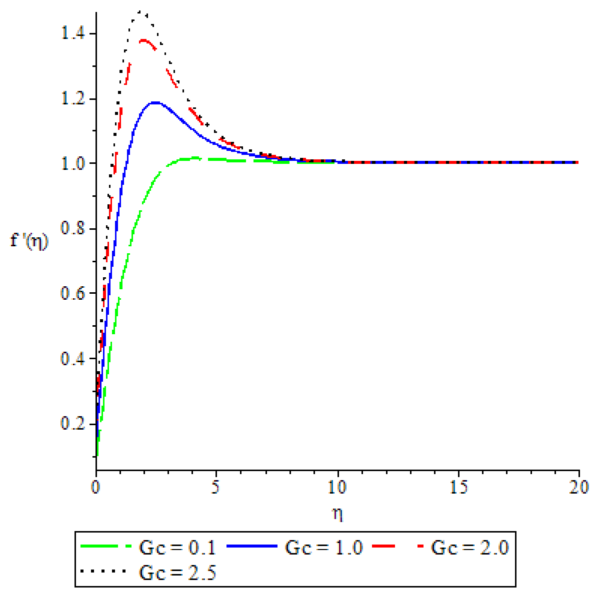

Fig. 3: Velocity profiles for varying Gc

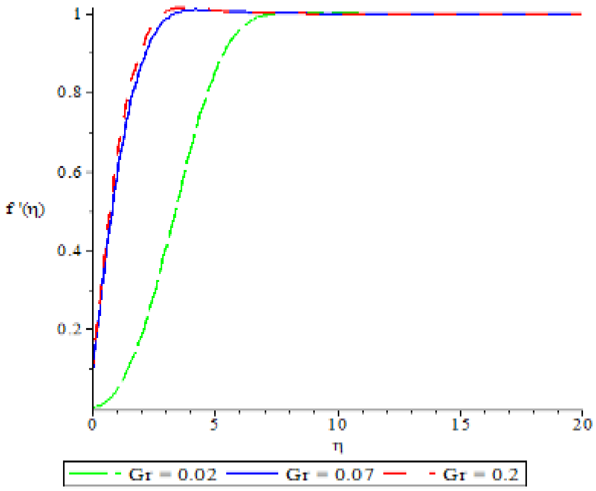

Fig. 4: Velocity profiles for varying Gr

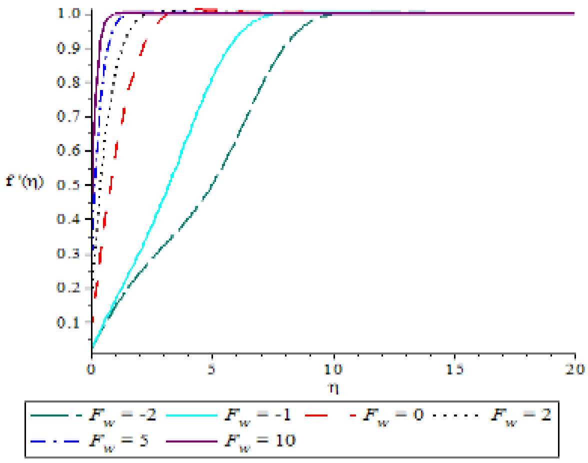

Fig. 5: Velocity profiles for varying F_W

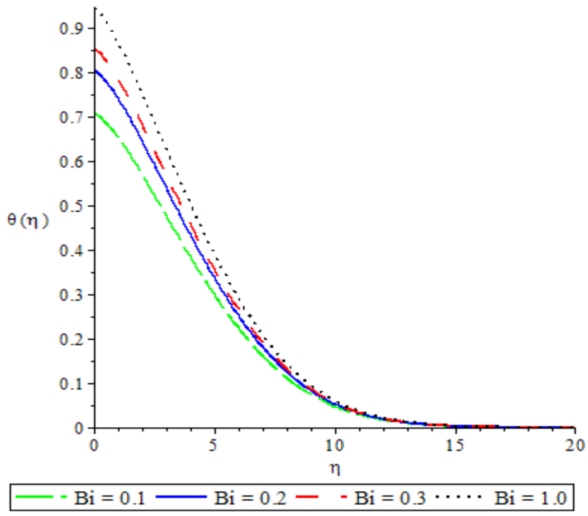

Fig. 6: Temperature profiles for varying Bi

Figures 6 – 13 show the effects of the emerging flow parameter on the temperature profile. The maximum value of the fluid temperature is attained at the plate surface but decreases to the free stream zero value away from the plate satisfying the boundary condition. The thermal boundary layer thickness increases with an increase in Biot number B_i. Due to increases in the heat transfer rate from the hot fluid at the lower side of the plate to the cold fluid at the upper side increases. Fig 7 shows the effect of Prandtl number on the temperature profile. The temperature profile decreases with increase in Prandtl number $\Pr$ because higher Prandtl number results in relatively low thermal conductivity on the fluid which reduces conduction and the thermal boundary layer thickness thereby decreasing the temperature. In Fig. 8, the thermal boundary layer thickness increases as the radiation parameter Ra increases. Also, in Fig. 9, increase in the magnetic parameter M increases the fluid temperature which in turn, increases the thermal boundary layer thickness due to ohmic heating on the flow system. In Fig. 10 and 11, progressive rise in fluid temperature and thickening of the thermal boundary layer were observed phenomena as Brinkman number B_r and internal heat generation parameter λ increases. Fluid suction makes the thermal boundary layer thinner in Fig. 12.

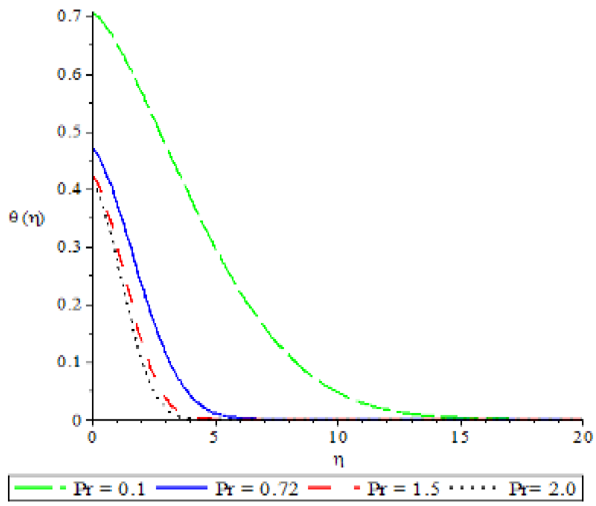

Fig. 7: Temperature profiles for varying $\Pr$

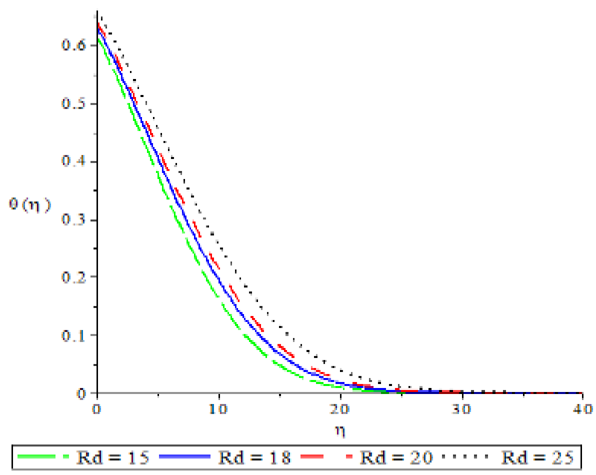

Fig. 8: Temperature profiles for varying Rd

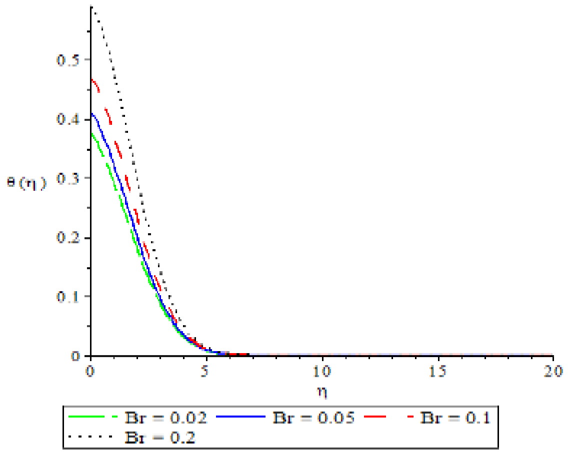

Fig. 9: Temperature profiles for varying Br

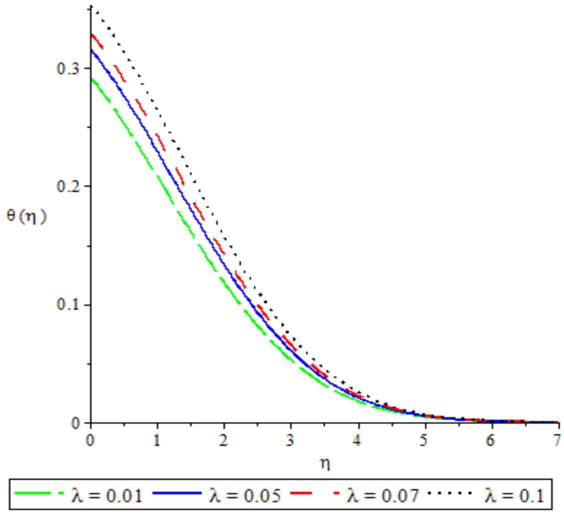

Fig. 10: Temperature profiles for varying λ

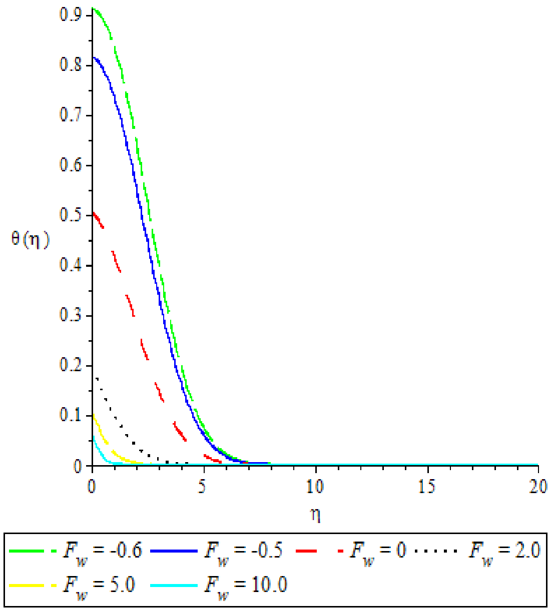

Fig. 11: Temperature profiles for varying F_W

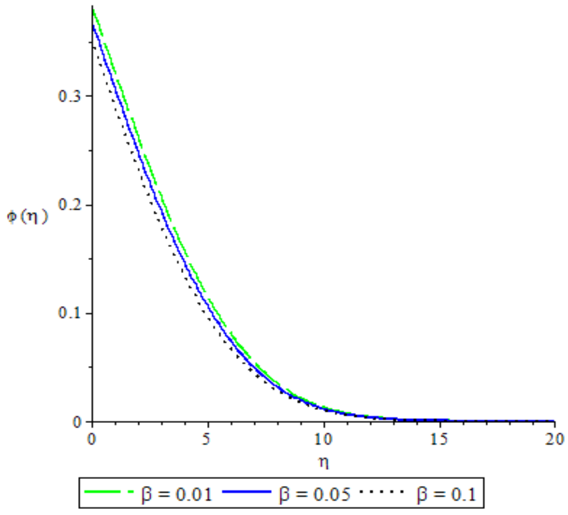

Fig. 12: Concentration profiles for varying β

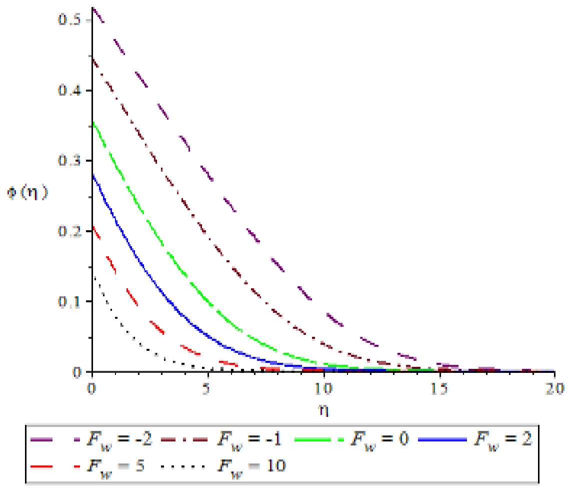

Fig. 13: Concentration profiles for varying F_W

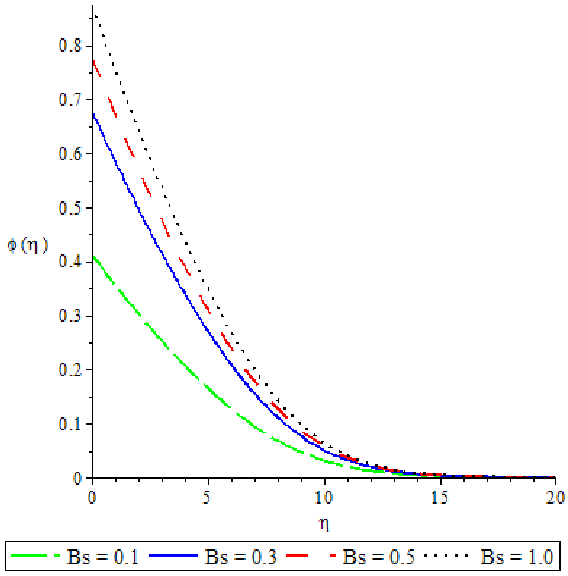

Fig. 14: Concentration profiles for varying Bs

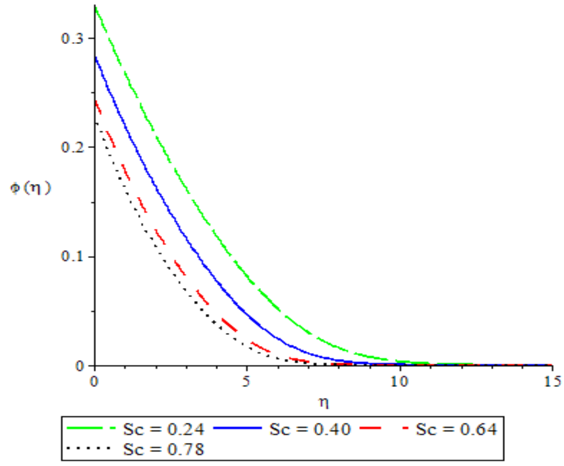

Fig. 15: Concentration profiles for varying Sc

In Figs. 13 – 14, concentration decreases with increase in the rate of chemical reaction parameter β and fluid Injection F_W >0 but increases with increase in the convective-diffusion parameter B_s. The boundary layer thickness also decreases for appreciable values of convective diffusion parameter Bs. The plate surface concentration increases with increase in convective diffusion parameter Bs but decreases with increase in the values of Schmidt number Sc, reaction rate parameter β and suction or injection F_W. In Fig. 15, the concentration level of the fluid drops due to increasing chemical reaction, fluid injection and Schmidt number because mass diffusivity raises the concentration level steadily.

Conclusions

From the numerical solutions and graphical representations, we can conclude that:

- The fluid velocity decreases at the plate and increases gradually to its free stream satisfying the boundary condition.
- The maximum value of the fluid temperature is attained at the plate surface but decreases to the free stream value of zero away from the plate satisfying the boundary condition.
- Increase in fluid suction retards the rate of transportation but decrease the boundary layer thickness and vice versa for fluid injection.
- The rate of heat transfer (from the plate to the fluid) decreases with thermal radiation and heat source parameter but increases with increase in Brinkman number, Eckert number, Pandtl number and ohmic heating of the flow system.
- Concentration of the flow decreases with increase in rate of chemical reaction and fluid injection but increases with increase in the convective-diffusion parameter.
- Increase in internal heat generation increase fluid temperature and thickening of the thermal boundary layer

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Conflict of Interest

The authors declare that there is no conflict of interest related to this study.

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